Effective method for coding RS codes using SIMD instructions

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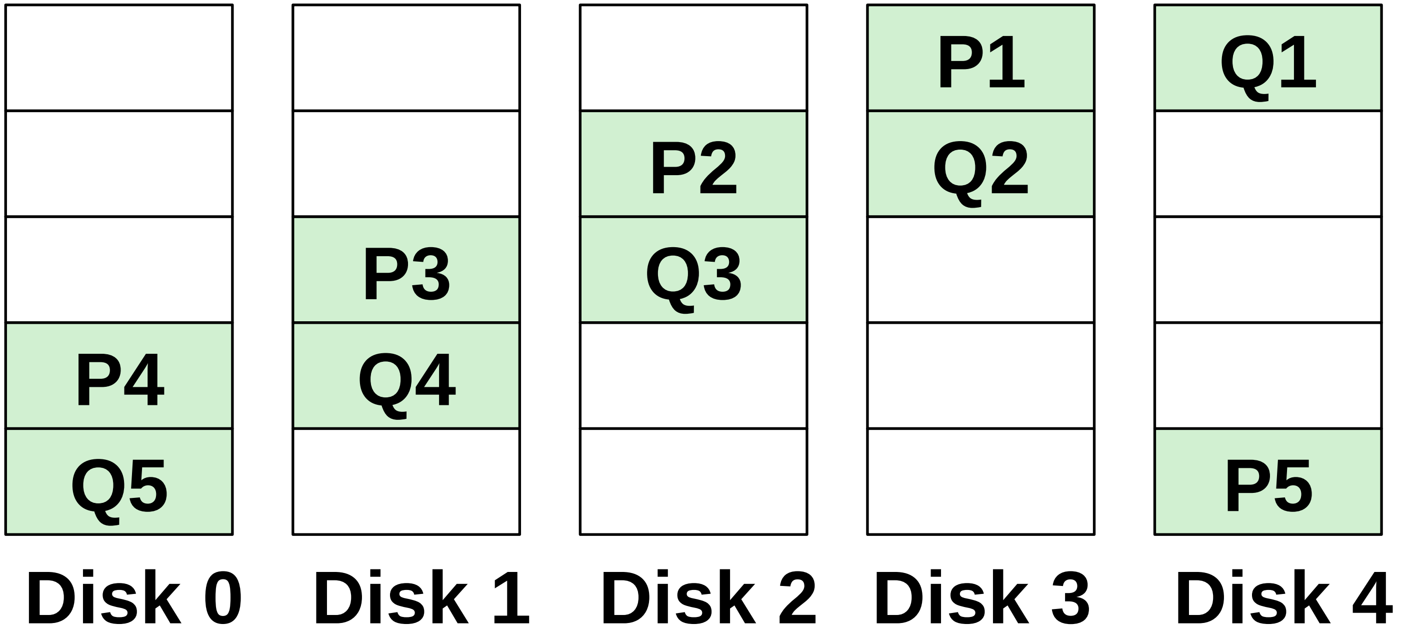
Outline

- Problem: Errors in data storage
- Method: Reed-Solomon codes
- Goal: Fast coding and Decoding

1. Background
    - Error types
    - Galois Fields arithmetic
    - Known solutions
2. Problem definition
    - Reed-Solomon Codes
3. Coding matrix
4. Recovery matrix
5. SDC detection
    - Main steps
6. Performance comparison

Error types



1. Failure – error at known positions
2. Silent Data Corruption (SDC) – error at unknown positions

$GF(2^q)$ Galois Fields Arithmetic

Operation with data blocks:

1. Addition
2. Multiplication
3. Inversion
4. Logarithm computation
5. etc.

Known Solutions

- RAID
- Local Reconstruction Codes (LRC) – Microsoft Azure
- Hadoop Distributed Raid File System
- Jerrasure
- Intel Storage Acceleration Library
- etc

Don’t solve the problem of several SDCs or SDCs + failures

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Problem statement

n data blocks $D_0, D_1, \dots, D_{n-1}$

$(D_0, D_1, \dots, D_{n-1}, C_0, C_1 \dots, C_{m-1})$ are stored to drives

- Calculation of m cheksums$C_0, C_1 \dots, C_{m-1}$
- Recovery of several failed drives
- Detection and recovery of several SDCs

Desire: High speed of coding and decoding

Reed-Solomon codes

$(D_0, D_1, \dots, D_{n-1}, C_0, C_1 \dots, C_{m-1})$ – RS code

$l =$ number of failures

$p =$ number of SDCs

If $p + 2l \leq m$ then all the errors can be corrected

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Coding via Coding Matrix

$$\begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ \vdots \\ C_{m-1} \end{pmatrix} = \begin{pmatrix} \widetilde{W}_1(a^{N-1}) & \widetilde{W}_1(a^{N-2}) & \cdots & \widetilde{W}_1(a^{m}) \\ \widetilde{W}_2(a^{N-1}) & \widetilde{W}_2(a^{N-2}) & \cdots & \widetilde{W}_2(a^{m}) \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{W}_m(a^{N-1}) & \widetilde{W}_m(a^{N-2}) & \cdots & \widetilde{W}_m(a^{m}) \end{pmatrix} \begin{pmatrix} D_0 \\ D_1 \\ D_2 \\ \vdots \\ D_{n-1} \end{pmatrix}$$

D_i - data blocks

C_j - checksums blocks

$\widetilde{W}_k$ - basic interpolation polynomials

$N = n + m$

a – primitive element of the field

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Notation for data recovery

$(Y_0, \ldots, Y_{N-1}) =$

$=(D_0, D_1, \ldots, D_{n-1}, C_0, \ldots, C_{m-1})$ - data and checksums blocks

$l =$ number of failed blocks $(l \leq m)$

$k_1, \ldots, k_l$ - positions of failures

$\bar{Y}_{l,1}$ - the column of failed blocks

$Y'_{(N-l),1}$ - the column of not failed blocks

Failures recovery via Recovery Matrix

$\bar{Y} = RY'$, $\bar{Y}$ - failed blocks, Y' - not failed blocks

$$R = \begin{pmatrix} \widetilde{W}_1(a^{N-1}) & \cdots & \widetilde{W}_1\left(a^{N-k_1}\right) & \widetilde{W}_1\left(a^{N-k_1-2}\right) & \cdots & \widetilde{W}_1(1) \\ \widetilde{W}_2(a^{N-1}) & \cdots & \widetilde{W}_2\left(a^{N-k_1}\right) & \widetilde{W}_2\left(a^{N-k_1-2}\right) & \cdots & \widetilde{W}_2(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{W}_l(a^{N-1}) & \cdots & \widetilde{W}_l\left(a^{N-k_1}\right) & \widetilde{W}_l\left(a^{N-k_1-2}\right) & \cdots & \widetilde{W}_l(1) \end{pmatrix}$$

- No extra memory for intermediate computations
- Simple Recovery matrix calculation
- Good parallelization

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SDC detection

$$(Y_0, \dots, Y_{N-1}) =$$

$= (D_0, D_1, \dots, D_{n-1}, C_0, C_1 \dots, C_{m-1})$ – RS code

$p =$ maximal number of SDCs ($\leq \lfloor m/2 \rfloor$)

$j_1, j_2, \dots, j_p$ - SDCs positions (a priori unknown)

Problem:

1. Detect the SDC presence
2. Find SDC positions

SDC detection

1. Syndrome calculation $S = VY$, where V – Vandermonde matrix
2. $S \rightarrow$ Berlekamp-Massey algorithm $\rightarrow$ error locator polynomial.
3. Error locator polynomial $\rightarrow$ Chien's Search $\rightarrow j_1, j_2, \dots, j_p$
4. Recovery via Recovery matrix

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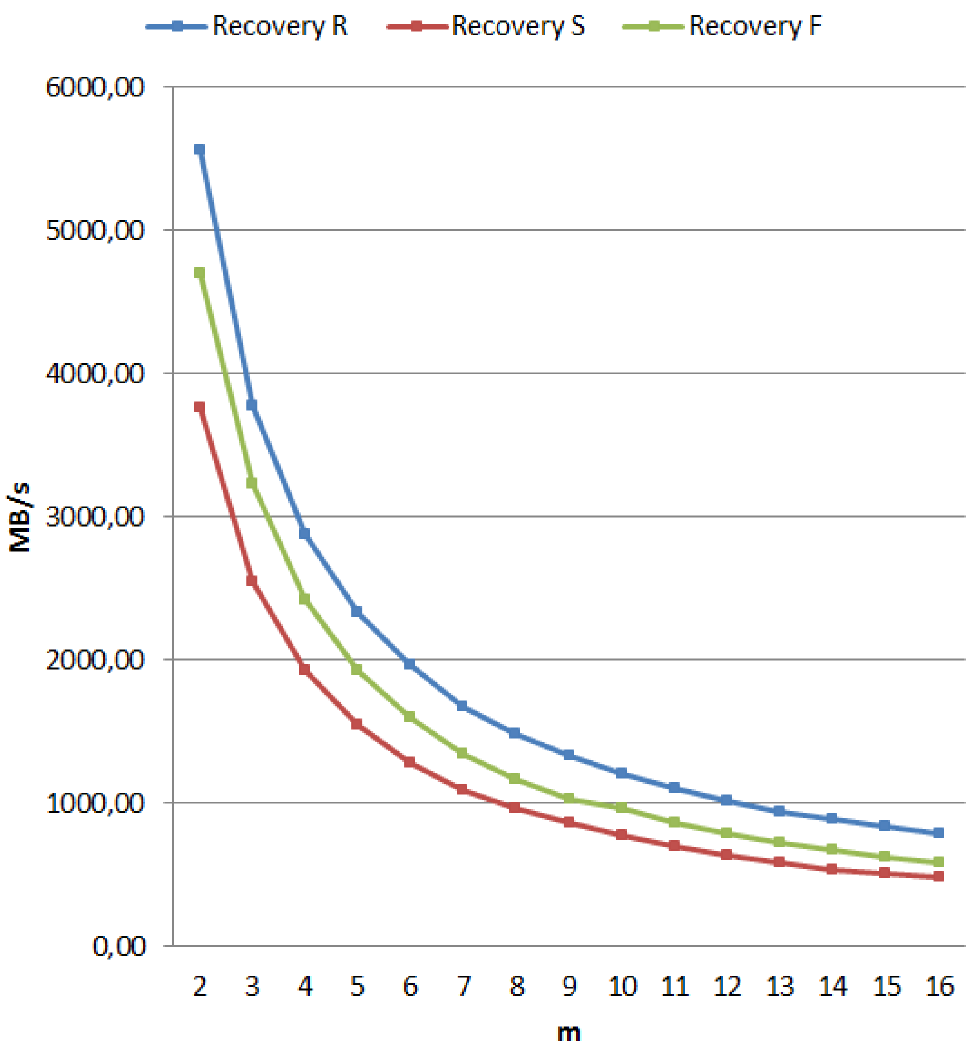
Recovery performance comparison



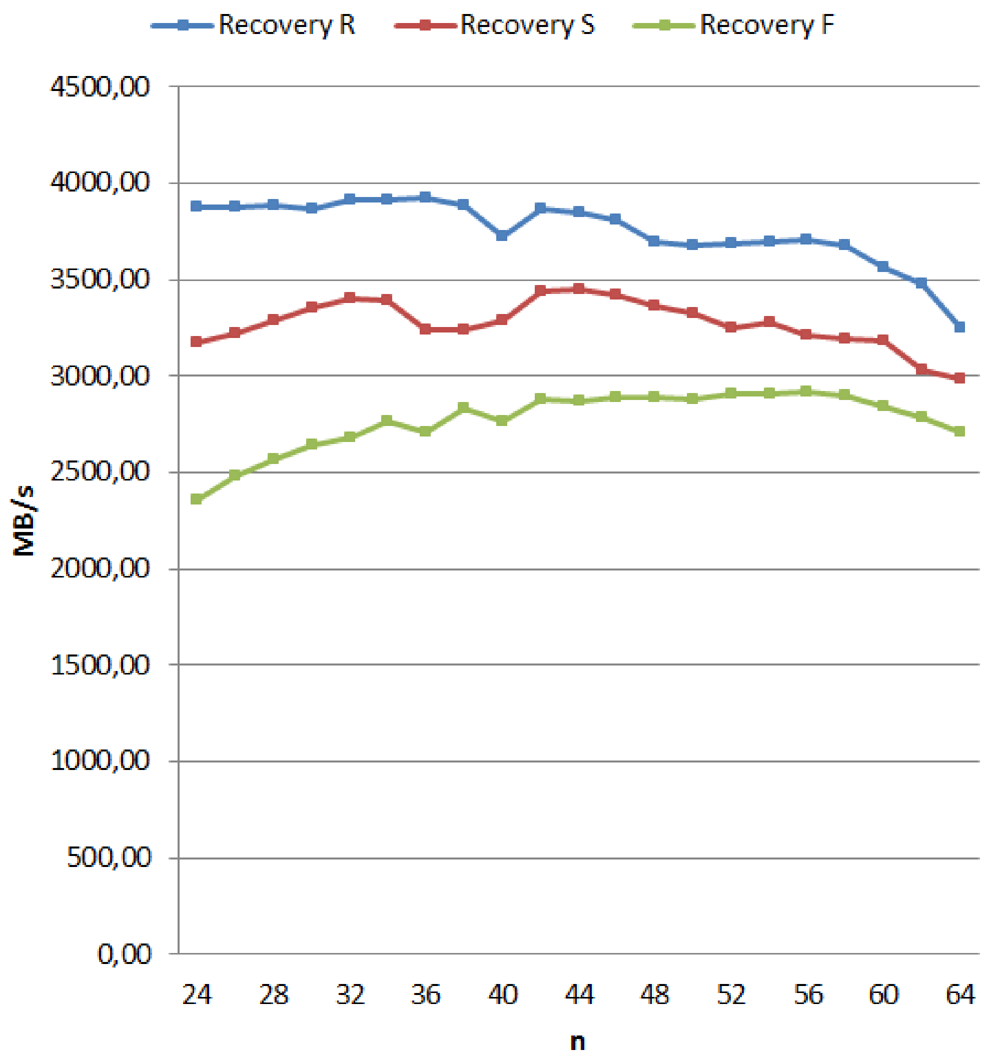
R – recovery matrix method

S – matrix inversion method

F – Forney Algorithm

- $n = 128$
- $m = 2 \ldots 16$
- Each block size = 4096 bytes

Recovery performance comparison



R – recovery matrix method

S – matrix inversion method

F – Forney Algorithm

- $n = 24 \ldots 64$
- $m = 4$
- Each block size = 4096 bytes

Conclusion

We suggest method for RS coding via matrix operations. Coding and Decoding up to 25 % faster.

Advantages

- Good parallelization
- Small memory requirements
- Strassen matrix multiplication algorithm

Thanks for your attention!

Questions please.

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Basic interpolation polynomials

Notation

$$W(\mathfrak{x}) = \prod_{i=1}^{n} (\mathfrak{x} - \lambda_i), \qquad W_j(\mathfrak{x}) = \frac{W(\mathfrak{x})}{\mathfrak{x} - \lambda_j}, \qquad j = \overline{1, n}$$

$$\widetilde{W}_j(\mathfrak{x}) = \frac{W_j(\mathfrak{x})}{W_j(\lambda_j)} = \frac{W_j(\mathfrak{x})}{W'(\lambda_j)}$$

$$= \frac{(\mathfrak{x} - \lambda_1)(\mathfrak{x} - \lambda_2) \dots (\mathfrak{x} - \lambda_{j-1})(\mathfrak{x} - \lambda_{j+1}) \dots (\mathfrak{x} - \lambda_n)}{(\lambda_j - \lambda_1)(\lambda_j - \lambda_2) \dots (\lambda_j - \lambda_{j-1})(\lambda_j - \lambda_{j+1}) \dots (\lambda_j - \lambda_n)} =$$

$$= w_{j,0} + w_{j,1}\mathfrak{x} + \cdots + w_{j,n-1}\mathfrak{x}^{n-1}$$

From Lagrange Interpolation

SDCs and Failures

$l =$ number of failures

$p =$ number of SDCs

$p + 2l \leq m$

1. Calculate syndromes $S = VY$
2. Construct polynomial

$$z(\mathfrak{x}) = \prod_{i=1}^{l}\left(\mathfrak{x} + a^{N-k_i-1}\right) = z_0 + z_1\mathfrak{x} + \cdots + z_l\mathfrak{x}^l$$

SDCs and Failures

3. Calculate

$$T_i = \sum_{j=0}^{l} z_j S_{i+j}, \qquad i = \overline{0, m-l-1}$$

4. T → BMA

5. Chien’s Search (to find SDC positions)

6. Recovery