Graphulo - TableMult

Server-side Sparse Matrix Multiply in the Accumulo Database

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This work is NOT
Creating the best system
for a particular task (matrix multiply)
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Creating the best system
for a particular task (matrix multiply)

This work IS
Adding graph analytic capabilities
(matrix multiply) to an all-around good
system used in practice today (Accumulo)
• Intro to Graphulo
• Intro to Matrix Multiply
• Intro to Accumulo
• Matrix Multiply pre-Graphulo
• Inner Product
• Outer Product
• Accumulo Implementation
• Performance
• Conclusions
Many groups store graph data in Accumulo

→ Need tools for graph analysis in Accumulo
Why Accumulo?

Accumulo ingest performance is 100x greater than competing technologies.
Graphulo Overview

• Primary Goal
  – Open source Apache Accumulo Java library that enables many graph algorithms in Accumulo

• Core primitives: GraphBLAS

• 3 Graph Schemas
  – Adjacency, Incidence, Single-Table

• 4 Demonstration Graph Algorithms
  – Degree-filtered Breadth First Search, Jaccard coefficients, k-Truss subgraph, Non-negative Matrix Factorization

• Focus on Interactive Computing
  – "Queued" / Localized analytics within a neighborhood, as opposed to whole table analytics
  – Low latency more important than high throughput
  – Progress monitoring for user sanity
    • Is the library working or stuck?
## GraphBLAS initial function list

<table>
<thead>
<tr>
<th>Function</th>
<th>Parameters</th>
<th>Returns</th>
<th>Math Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SpGEMM</td>
<td>- sparse matrices $A$ and $B$  &lt;br&gt; - unary functors (op)</td>
<td>sparse matrix</td>
<td>$C = \text{op}(A) \times \text{op}(B)$</td>
</tr>
<tr>
<td>SpM{Sp}V</td>
<td>- sparse matrix $A$  &lt;br&gt; - sparse/dense vector $x$</td>
<td>sparse/dense vector</td>
<td>$y = A \times x$</td>
</tr>
<tr>
<td></td>
<td>(Sp: sparse)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SpEWiseX</td>
<td>- sparse matrices or vectors  &lt;br&gt; - binary functor and predicate</td>
<td>in place or sparse</td>
<td>$C = A \times B$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>matrix/vector</td>
<td></td>
</tr>
<tr>
<td>Reduce</td>
<td>- sparse matrix $A$ and functors</td>
<td>dense vector</td>
<td>$y = \text{sum}(A, \text{op})$</td>
</tr>
<tr>
<td>SpRef</td>
<td>- sparse matrix $A$  &lt;br&gt; - index vectors $p$ and $q$</td>
<td>sparse matrix</td>
<td>$B = A(p,q)$</td>
</tr>
<tr>
<td>SpAsgn</td>
<td>- sparse matrices $A$ and $B$  &lt;br&gt; - index vectors $p$ and $q$</td>
<td>none</td>
<td>$A(p,q) = B$</td>
</tr>
<tr>
<td>Scale</td>
<td>- sparse matrix $A$  &lt;br&gt; - dense matrix or vector $X$</td>
<td>none</td>
<td>check manual</td>
</tr>
<tr>
<td>Apply</td>
<td>- any matrix or vector $X$  &lt;br&gt; - unary functor (op)</td>
<td>none</td>
<td>$\text{op}(X)$</td>
</tr>
</tbody>
</table>
Outline

• Intro to Graphulo
• Intro to Matrix Multiply
• Intro to Accumulo
• Matrix Multiply pre-Graphulo
• Inner Product
• Outer Product
• Accumulo Implementation
• Performance
• Conclusions
Matrix Multiply on Big Data

Traditional Matrix Multiply: $AB = C$

\[
\begin{bmatrix}
6 & 5 & 0 & 2 \\
0 & 4 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
0 & 3 \\
5 & 0 \\
3 & 4
\end{bmatrix} =
\begin{bmatrix}
6 & 23 \\
0 & 12
\end{bmatrix}
\]
Matrix Multiply on Big Data

Traditional Matrix Multiply: $AB = C$

$$\begin{bmatrix} 6 & 5 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 3 \\ 5 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 23 \\ 0 & 12 \end{bmatrix}$$

Row & Column Labels

Database Table Multiply

$$\begin{bmatrix} \text{word|coffee} & \text{word|dew} & \text{word|hot} \\ \text{tod|0500} & \text{tod|0800} & \text{tod|0900} & \text{tod|1400} \\ \text{word|coffee} & \text{word|dew} & \text{word|hot} \\ \text{tod|0500} & \text{tod|0800} & \text{tod|0900} & \text{tod|1400} \end{bmatrix} = \begin{bmatrix} 6 & 23 \\ 0 & 12 \end{bmatrix}$$
Matrix Multiply on Big Data

Traditional Matrix Multiply: \( AB = C \)

\[
\begin{bmatrix}
6 & 5 & 0 & 2 \\
0 & 4 & 0 & 0 \\
3 & 4 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
0 & 3 \\
5 & 0 \\
3 & 4
\end{bmatrix} =
\begin{bmatrix}
6 & 23 \\
0 & 12
\end{bmatrix}
\]

- Row & Column Labels
- Sparse

Database Table Multiply

\[
\begin{array}{cc}
\text{word} & \text{coffee} \\
\text{word} & \text{desert} \\
\text{tod|0500} & 6 \\
\text{tod|0800} & 5 \\
\text{tod|1400} & 2
\end{array}
\begin{array}{cc}
\text{word} & \text{dew} \\
\text{word} & \text{hot} \\
\text{tod|0800} & 3 \\
\text{tod|0900} & 5 \\
\text{tod|1400} & 3
\end{array}
\begin{array}{cc}
\text{word} & \text{coffee} \\
\text{word} & \text{dew}
\end{array}
\begin{array}{cc}
\text{word} & \text{dew} \\
\text{word} & \text{hot}
\end{array}
\]

\[
\begin{bmatrix}
6 & 23 \\
0 & 12
\end{bmatrix}
\]
Matrix Multiply on Big Data

Traditional Matrix Multiply: $AB = C$

$$\begin{bmatrix} 6 & 5 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 3 \\ 5 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 23 \\ 0 & 12 \end{bmatrix}$$

- Row & Column Labels
- Sparse
- Associative Array

Mathematics

Database Table Multiply

<table>
<thead>
<tr>
<th>word</th>
<th>coffee</th>
<th>deck</th>
<th>hot</th>
<th>dew</th>
<th>word</th>
<th>dew</th>
<th>hot</th>
</tr>
</thead>
<tbody>
<tr>
<td>tod</td>
<td>0500</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>word</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>tod</td>
<td>0800</td>
<td>4</td>
<td></td>
<td></td>
<td>word</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>tod</td>
<td>1400</td>
<td></td>
<td></td>
<td></td>
<td>word</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\begin{bmatrix} 6 & 5 \\ 0 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 23 \\ 3 & 12 \end{bmatrix}$

---

Application: Multi-Source Breadth-First Search

- Sparse array representation => space efficient
- Sparse matrix-matrix multiplication => work efficient
- Three possible levels of parallelism: searches, vertices, edges
- Basis for a wide range of graph algorithms
Application: Multi-Source Breadth-First Search

- Sparse array representation => space efficient
- Sparse matrix-matrix multiplication => work efficient
- Three possible levels of parallelism: searches, vertices, edges
- Basis for a wide range of graph algorithms
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Background on Accumulo

Best for:

- Large, de-normalized tables (NoSQL)
- Hadoop HDFS / Java ecosystem
- Huge data volume – TBs to PBs
- Cell-level visibility
- Robust horizontal scaling

- Row store by default
  - Scan over rows for $O(\log n)$ lookup & sorted order
  - Log-structured Merge Tree design
- Iterator processing framework

Use Transpose Tables
see D4M Schema¹
Background on Accumulo

Best for:
- Large, de-normalized tables (NoSQL)
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- Row store by default
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Use Transpose Tables
see D4M Schema

1 D4M 2.0 Schema: A General Purpose High Performance Schema for the Accumulo Database
Kepner et al, IEEE HPEC 2013
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Table Multiply Before Graphulo

Client

Accumulo

A

B
Table Multiply Before Graphulo

Graphulo - TableMul-21
Table Multiply Before Graphulo

Multiply in-memory*

*A blocked algorithm exists for large tables at reduced efficiency.
Table Multiply Before Graphulo

A  B  C
Client

A  B  C
Accumulo

Write
Table Multiply Before Graphulo

Old: $DB = \text{Indexed Storage}$

*Blocked algorithms exist for large tables at reduced efficiency
Table Multiply Before Graphulo

Old:  DB = Indexed Storage
New:  DB = Indexed Storage + Computation Engine

*Blocked algorithms exist for large tables at reduced efficiency
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Inner Product

\[
\begin{bmatrix}
\text{word|coffee} & \text{word|dew} & \text{word|hot} \\
6 & 5 & 2 \\
0 & 0 & 0 \\
0 & 4 & 0 \\
0 & 0 & 3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{tod|0500} & \text{tod|0800} & \text{tod|1400} \\
6 & 5 & 2 \\
3 & 3 & 4 \\
\end{bmatrix}
\]

\[
\begin{align*}
\text{for } i &= 1: N = 2 \\
\text{for } j &= 1: L = 2 \\
\text{for } k &= 1: M = 4 \\
C(i, j) &= \bigoplus_{k=1}^{M} A(i, k) \otimes B(k, j)
\end{align*}
\]
Inner Product

\[
\begin{bmatrix}
6 & 5 & 2 \\
4 & 3 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
5 & 3 & 3 \\
4 & 0 & 0 \\
2 & 0 & 0 \\
\end{bmatrix}
= 
\begin{bmatrix}
1 & 6 & 23 \\
2 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\text{for } i = 1: N = 2  \\
\quad \text{for } j = 1: L = 2  \\
\quad \quad \text{for } k = 1: M = 4  \\
C(i, j) \oplus A(i, k) \otimes B(k, j)
\]
Inner Product

\[
\begin{bmatrix}
\text{word|coffee} & 6 & 5 & 2 \\
\text{word|desert} & 4 & 3 & 0 \\
\end{bmatrix}_{\text{tod|0500}}
\begin{bmatrix}
\text{word|dew} & 3 \\
\text{word|hot} & 4 \\
\end{bmatrix}_{\text{tod|0800}} =
\begin{bmatrix}
\text{word|coffee} & 6 \\
\text{word|dew} & 23 \\
\end{bmatrix}_{\text{tod|1400}}
\]

\[
\text{for } i = 1: N = 2 \\
\text{for } j = 1: L = 2 \\
\text{for } k = 1: M = 4 \\
C(i, j) \oplus= A(i, k) \otimes B(k, j)
\]

\[
C(i, j) = \bigoplus_{k=1}^{M} A(i, k) \otimes B(k, j)
\]
### Inner Product

**Graphulo Table Multi**

| Word   | tod|0500 | tod|0800 | tod|1400 |
|--------|----|------|------|------|------|
| coffee | 6  | 5    | 2    |      |      |
| desert | 4  |      |      |      |      |

| Word   | tod|0800 | tod|0900 | tod|1400 |
|--------|----|------|------|------|------|
| dew    | 3  | 5    | 3    |      |      |
| hot    | 4  |      |      |      |      |

\[
C(i, j) = \bigoplus_{k=1}^{M} A(i, k) \otimes B(k, j)
\]

**2nd Scan**

**for** \( i = 1 : N = 2 \)

**for** \( j = 1 : L = 2 \)

**for** \( k = 1 : M = 4 \)

\[
C(i, j) \oplus= A(i, k) \otimes B(k, j)
\]
Inner Product

+ Write locality (sorted)
+ Pre-sum partial products
  (3 entries written)
– N scans over table B

\[
\begin{align*}
\text{word|coffee} & \quad \text{word|desert} \\
\text{tod|0500} & \quad \text{5} \\
\text{tod|0800} & \quad \text{6} \\
\text{tod|1400} & \quad \text{2}
\end{align*}
\]

\[
\begin{align*}
\text{word|dew} & \quad \text{word|hot} \\
\text{tod|0800} & \quad \text{3} \\
\text{tod|0900} & \quad \text{4} \\
\text{tod|1400} & \quad \text{2}
\end{align*}
\]

\[
\begin{align*}
\text{word|dew} & \quad \text{word|hot} \\
\text{tod|0800} & \quad \text{5} \\
\text{tod|0900} & \quad \text{6} \\
\text{tod|1400} & \quad \text{12}
\end{align*}
\]

\[
\begin{align*}
\text{word|dew} & \quad \text{word|hot} \\
\text{tod|0800} & \quad \text{3} \\
\text{tod|0900} & \quad \text{4} \\
\text{tod|1400} & \quad \text{23}
\end{align*}
\]

\[
C(i, j) = \bigoplus_{k=1}^{M} A(i, k) \otimes B(k, j)
\]

\[
\begin{align*}
\text{for } i = 1: N = 2 \\
\text{for } j = 1: L = 2 \\
\text{for } k = 1: M = 4
\end{align*}
\]

\[
\begin{align*}
\text{for } k = 1: M = 4
\end{align*}
\]

\[
\text{C}(i, j) \oplus= A(i, k) \otimes B(k, j)
\]

\[
\begin{align*}
\text{for } i = 1: N = 2 \\
\text{for } j = 1: L = 2 \\
\text{for } k = 1: M = 4
\end{align*}
\]
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Outer Product

Now explicitly showing $A^T$

\[
\begin{array}{ccc}
\text{tod|0500} & 6 & 3 \\
\text{tod|0800} & 5 & 3 \\
\text{tod|1400} & 2 & 3 \\
\end{array}
\begin{array}{ccc}
\text{word|coffee} \\
\text{word|desert} \\
\text{word|dew} \\
\end{array}
\begin{array}{ccc}
\text{word|hot} \\
\end{array}
\]

\[
\begin{align*}
\text{tod|0500} & \begin{bmatrix} 6 \\ 5 \\ 2 \end{bmatrix} \\
\text{tod|0800} & \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \\
\text{tod|1400} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
&= \\
&= \\
\end{align*}
\]

\[
\begin{array}{cccc}
\text{tod|0500} & 6 & 3 & 0 \\
\text{tod|0800} & 5 & 3 & 0 \\
\text{tod|1400} & 2 & 3 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{word|coffee} & \text{word|dew} & \text{word|hot} \\
\text{word|dew} & \text{word|hot} \\
\text{word|hot} & \\
\end{array}
\]

\[
\begin{align*}
&\text{for } k = 1: M = 4 \\
&\text{for } i = 1: N = 2 \\
&\text{for } j = 1: L = 2 \\
&\quad C(i, j) \oplus A(i, k) \otimes B(k, j)
\end{align*}
\]

\[
C = \bigoplus_{k=1}^{M} A(:, k)B(k, :)
\]
# Outer Product

## 1. Align Rows

<table>
<thead>
<tr>
<th>time</th>
<th>word</th>
<th>coffee</th>
<th>desert</th>
<th>dew</th>
<th>hot</th>
</tr>
</thead>
<tbody>
<tr>
<td>tod</td>
<td>0500</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>tod</td>
<td>0800</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>tod</td>
<td>1400</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ \begin{bmatrix} 6 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

\[
\begin{align*}
\text{for } & \ k = 1: M = 4 \\
\text{for } & \ i = 1: N = 2 \\
\text{for } & \ j = 1: L = 2 \\
C(i, j) \oplus A(i, k) \otimes B(k, j)
\end{align*}
\]
### 1. Align Rows

<table>
<thead>
<tr>
<th>tod</th>
<th>0500</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>tod</td>
<td>0800</td>
<td>5</td>
</tr>
<tr>
<td>tod</td>
<td>1400</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>tod</th>
<th>0800</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>tod</td>
<td>0900</td>
<td>5</td>
</tr>
<tr>
<td>tod</td>
<td>1400</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
C = \bigoplus_{k=1}^{M} A(\cdot, k) B(k, \cdot)
\]

**Algorithm**

\[
\begin{align*}
&\text{for } k = 1 : M = 4 \\
&\quad\text{for } i = 1 : N = 2 \\
&\quad\quad\text{for } j = 1 : L = 2 \\
&\quad\quad\quad C(i, j) \oplus = A(i, k) \otimes B(k, j)
\end{align*}
\]
2. Cartesian Product

\[
\begin{align*}
\text{for } k = 1: M &= 4 \\
\text{for } i = 1: N &= 2 \\
\text{for } j = 1: L &= 2 \\
C(i, j) \odot &= A(i, k) \otimes B(k, j)
\end{align*}
\]

\[
C = \bigoplus_{k=1}^{M} A(:, k)B(k, :)
\]
Outer Product

2. Cartesian Product

\[
\begin{array}{c|c|c}
\text{tod|0500} & 6 & 4 \\
\text{tod|0800} & 5 \\
\text{tod|1400} & 2 \\
\end{array}
\quad \begin{array}{c|c|c}
\text{tod|0800} & 3 \\
\text{tod|0900} & 5 \\
\text{tod|1400} & 4 \\
\end{array}
\]

\[C = \bigoplus_{k=1}^{M} A(:,k) \otimes B(k,:)\]

\text{for } k = 1: M = 4
\text{for } i = 1: N = 2
\text{for } j = 1: L = 2
\quad C(i,j) \oplus A(i,k) \otimes B(k,j)
## Outer Product

### 1. Align Rows

<table>
<thead>
<tr>
<th>word</th>
<th>coffee</th>
<th>word</th>
<th>desert</th>
<th>word</th>
<th>dew</th>
<th>word</th>
<th>hot</th>
</tr>
</thead>
<tbody>
<tr>
<td>tod</td>
<td>0500</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tod</td>
<td>0800</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tod</td>
<td>1400</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>tod</td>
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<td></td>
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<td>5</td>
<td></td>
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<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

\[
C = \bigoplus_{k=1}^{M} A(:, k) \otimes B(k, :) 
\]

**for** \( k = 1 : M = 4 \)

**for** \( i = 1 : N = 2 \)

**for** \( j = 1 : L = 2 \)

\( C(i, j) \oplus A(i, k) \otimes B(k, j) \)
Outer Product

1. Align Rows

\[
\begin{align*}
\text{word|coffee} & \quad \text{word|desert} & \quad \text{word|dew} & \quad \text{word|hot} \\
\text{tod|0500} & \quad 6 & \quad 4 & \\
\text{tod|0800} & \quad 5 & \quad 4 & \\
\text{tod|1400} & \quad 2 & \quad 3 & \quad 4 \\
\end{align*}
\]

\[
\begin{align*}
\text{word|coffee} & \quad \text{word|desert} \\
\text{tod|0800} & \quad 3 \\
\text{tod|0900} & \quad 5 \\
\text{tod|1400} & \quad 3 & \quad 4 \\
\end{align*}
\]

\[
C = \bigoplus_{k=1}^{M} A(:, k) B(k, :)
\]

for \( k = 1 : M = 4 \) 
\begin{align*}
\text{for } i = 1 : N = 2 \\
\text{for } j = 1 : L = 2 \\
C(i, j) \oplus A(i, k) \otimes B(k, j)
\end{align*}
### Outer Product

#### 2. Cartesian Product

\[ \text{for } k = 1: M = 4 \]
\[ \text{for } i = 1: N = 2 \]
\[ \text{for } j = 1: L = 2 \]
\[ C(i, j) \oplus A(i, k) \otimes B(k, j) \]

\[
\begin{bmatrix}
6 & 0 & 5 & 0 \\
5 & 0 & 4 & 0 \\
2 & 0 & 3 & 0 \\
6 & 0 & 3 & 4 \\
\end{bmatrix} =
\begin{bmatrix}
word|coffee \\
word|desert \\
word|dew \\
word|hot \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
6 & 0 & 5 & 0 \\
5 & 0 & 4 & 0 \\
2 & 0 & 3 & 0 \\
6 & 0 & 3 & 4 \\
\end{bmatrix} =
\begin{bmatrix}
word|coffee \\
word|dew \\
word|hot \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
15 & 0 & 12 \\
15 & 0 & 12 \\
\end{bmatrix}
\]

\[
C = \bigoplus_{k=1}^{M} A(:, k) \otimes B(k, :)
\]
**Outer Product**

### 2. Cartesian Product

| tod|0500 | tod|0800 | tod|1400 |
|----|------|------|------|
| word|coffee | word|coffee | word|coffee |
| 6   | 5     | 2    |
| word|desert | word|desert | word|desert |
| 4   | 4     | 4    |
| tod|0800 | tod|0900 | tod|1400 |
| 5   | 5     | 3    |
| word|dew   | word|dew   | word|dew   |
| 3   | 3     | 4    |

\[ \text{word|coffee} \times \text{word|desert} = \begin{bmatrix} 6 & 23 \\ 12 & * \end{bmatrix} \]

\[ \text{word|dew} \times \text{word|hot} \]

\[ C = \bigoplus_{k=1}^{M} A(:, k) \otimes B(k, :) \]

\[ \text{for } k = 1: M = 4 \]

\[ \text{for } i = 1: N = 2 \]

\[ \text{for } j = 1: L = 2 \]

\[ C(i, j) \oplus A(i, k) \otimes B(k, j) \]

*Lazy \( \oplus \): Accumululo stores both 15 and 8 until next scan or compaction*
Outer Product

- No write locality; unsorted writes
- Hard to pre-sum partial products
  (4 entries written)

+ Single scan over table B

\[
\begin{pmatrix}
6 & 5 & 2 \\
5 & 4 & \text{tod|0800} \\
2 & \text{tod|1400}
\end{pmatrix}
\]

\[
\begin{pmatrix}
3 & 5 & 3 \\
3 & 4 & \text{tod|1400}
\end{pmatrix}
\]

\[
\begin{pmatrix}
6 & 23 & 1 \\
12 & 4 & \text{tod|1400}
\end{pmatrix}
\]

\[
C = \bigoplus_{k=1}^{M} A(:,k) \otimes B(k,:)
\]

Graphulo-TableMult-42
Inner vs. Outer Product

• Outer product best for Accumulo
  – Single pass over table B = single disk read
  – BatchWriter ingest handles unsorted writes
  – Combiners handle $\oplus$
  – Less extra partial products written for sparse data

• Inner product still has merit
  – Better for dense data
  – Hybrid 2D-like algorithm possible
Outline

- Intro to Graphulo
- Intro to Matrix Multiply
- Intro to Accumulo
- Matrix Multiply pre-Graphulo
- Inner Product
- Outer Product
- Accumulo Implementation
- Performance
- Conclusions
Outer Product in Graphulo Iterators
Accumulo Distributes Graphulo Iterators

- Tablets can be hosted on any tablet server
  - Accumulo load balances tablet allocation
- Matrix multiply iterators run on B's tablets in parallel
  - Scan from A's tablets in parallel
  - BatchWrite to C's tablets in parallel

Key
IMM: In-Memory Map
RFILE: Hadoop File
Outline

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• Intro to Matrix Multiply
• Intro to Accumulo
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• Outer Product
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Performance Experiment

• Compare to pre-Graphulo alternative:
  – D4M Matlab client as Middleman
• Scaled / Weak scaling study:
  – How multiply rate varies with increasing problem size at fixed resources
  – Ideal: constant multiply rate
• Fixed / Strong scaling study:
  – How multiply rate varies with increasing resources at fixed problem size
  – Ideal: multiply rate scales linearly with increasing resources

• Environment:
  – Laptop, 16GB RAM, 2 Dual-core i7 processors, Accumulo 1.6.1
• Vary problem size between SCALE 10 and 18
  – Unpermuted Power law graph generator
  – # of nodes in each input table is $2^{\text{SCALE}}$. Used 16 edges/node
• Vary resources with # Accumulo Tablets (Varies # Threads)
Performance Experiment

TableMult Rate Scaling

- Graphulo 1 Tablet
- Graphulo 2 Tablets
- D4M 1 Tablet
- D4M 2 Tablets

Rate (partial products/s)

SCALE

10 11 12 13 14 15 16 17 18

$4 \times 10^5$
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Conclusion

• Promising performance
  – Write rates near 400k / sec, near highest single-node recorded rates
  – Experiments on a larger cluster will confirm weak & strong scaling

• Outer product better suited to Accumulo
  – Hybrid inner-outer product algorithms worth studying

• Current Graphulo research is
  – implementing remaining GraphBLAS
  – developing graph algorithms
### TABLE I: Output Table C Sizes and Experiment Timings

<table>
<thead>
<tr>
<th>SCALE</th>
<th>Entries in Table C</th>
<th>Graphulo 1 Tablet</th>
<th>D4M 1 Tablet</th>
<th>Graphulo 2 Tablets</th>
<th>D4M 2 Tablets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PartialProducts</td>
<td>AfterSum</td>
<td>Time (s)</td>
<td>Rate (pp/s)</td>
<td>Time (s)</td>
</tr>
<tr>
<td>10</td>
<td>$8.05 \times 10^5$</td>
<td>$2.69 \times 10^5$</td>
<td>2.87</td>
<td>$2.81 \times 10^5$</td>
<td>3.02</td>
</tr>
<tr>
<td>11</td>
<td>$2.36 \times 10^6$</td>
<td>$8.15 \times 10^5$</td>
<td>7.76</td>
<td>$3.04 \times 10^5$</td>
<td>8.80</td>
</tr>
<tr>
<td>12</td>
<td>$6.82 \times 10^6$</td>
<td>$2.43 \times 10^6$</td>
<td>2.20 $10^1$</td>
<td>$3.10 \times 10^5$</td>
<td>2.66 $10^1$</td>
</tr>
<tr>
<td>13</td>
<td>$1.91 \times 10^7$</td>
<td>$7.04 \times 10^6$</td>
<td>6.40 $10^1$</td>
<td>$2.99 \times 10^5$</td>
<td>1.50 $10^2$</td>
</tr>
<tr>
<td>14</td>
<td>$5.27 \times 10^7$</td>
<td>$2.00 \times 10^7$</td>
<td>1.82 $10^2$</td>
<td>$2.90 \times 10^5$</td>
<td>5.79 $10^2$</td>
</tr>
<tr>
<td>15</td>
<td>$1.47 \times 10^8$</td>
<td>$5.83 \times 10^7$</td>
<td>5.03 $10^2$</td>
<td>$2.93 \times 10^5$</td>
<td>2.51 $10^3$</td>
</tr>
<tr>
<td>16</td>
<td>$4.00 \times 10^8$</td>
<td>$1.63 \times 10^8$</td>
<td>1.39 $10^3$</td>
<td>$2.88 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>$1.09 \times 10^9$</td>
<td>$4.59 \times 10^8$</td>
<td>4.06 $10^3$</td>
<td>$2.67 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>$2.94 \times 10^9$</td>
<td>$1.28 \times 10^9$</td>
<td>1.21 $10^4$</td>
<td>$2.42 \times 10^5$</td>
<td></td>
</tr>
</tbody>
</table>
Inner-Outer Hybrid Algorithm

\[
\text{for } p = 1 : P \\
\text{for } k = 1 : M \\
\text{for } i = \left( \left\lfloor \frac{(p-1)N}{P} \right\rfloor + 1 \right) : \left\lfloor \frac{pN}{P} \right\rfloor \\
\text{for } j = 1 : L \\
C(i, j) \oplus= A(i, k) \otimes B(k, j)
\]

\[P = N \quad \text{– Inner Product}\]
\[P = 1 \quad \text{– Outer Product}\]
**D4M Schema for Sparse Arrays in Key/Value Databases (Accumulo)**

**Input Data**

<table>
<thead>
<tr>
<th>Time</th>
<th>Col1</th>
<th>Col2</th>
<th>Col3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001-01-01</td>
<td>a</td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>2001-01-02</td>
<td>b</td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>2001-01-03</td>
<td>c</td>
<td>c</td>
<td></td>
</tr>
</tbody>
</table>

**Accumulo Table:**

<table>
<thead>
<tr>
<th></th>
<th>01-01-2001</th>
<th>02-01-2001</th>
<th>03-01-2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Col1</td>
<td>a</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Col1</td>
<td>b</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Col2</td>
<td>b</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Col2</td>
<td>c</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Col3</td>
<td>a</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Col3</td>
<td>c</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Accumulo Table:**

<table>
<thead>
<tr>
<th>Col1</th>
<th>a</th>
<th>Col1</th>
<th>b</th>
<th>Col2</th>
<th>b</th>
<th>Col2</th>
<th>c</th>
<th>Col3</th>
<th>a</th>
<th>Col3</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-01-2001</td>
<td>1</td>
<td>02-01-2001</td>
<td>1</td>
<td>03-01-2001</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Tabular data expanded to create many type/value columns
- Transpose pairs allows quick look up of either row or column

---

1. D4M 2.0 Schema: A General Purpose High Performance Schema for the Accumulo Database
Kepner et al, IEEE HPEC 2013